Mathematical Fundamentals of Volatility Futures

David H. Annis, Ph.D.
Vernon Capital Partners, LLC.
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AN PRIMER ON VOLATILITY

Volatility is a statistical measure of dispersion. In financial markets, it is used to quantify the degree of uncertainty associated with the returns of an asset. Higher levels of volatility correspond to higher levels of uncertainty and larger price movements. Conversely, lower levels of volatility correspond to lower levels of uncertainty and more gradual price changes. While standard deviation as a statistical measure is well-defined, in a financial sense, volatility is model-dependent insofar as the calculation of dispersion depends on, among other things, the data considered, the denomination of the payoff and the cost to finance the asset. For example, the standard deviation of price returns will not be the same as the standard deviation of log-returns. Likewise, the standard deviation of the price of a U.S. Dollar-denominated asset will not be the same as the same asset denominated in Swiss Francs.

REALIZED VOLATILITY

Realized volatility is numerical quantification of the variability of asset returns, expressed, by convention, as an annualized standard deviation of (often logarithmic) returns. Although the term *realized* volatility may be used interchangeably with *historical* volatility, the latter implies an unnecessary retrospective context. Figure 1 illustrates the notion of realized volatility using the price history of the S&P 500 index and an artificial low-volatility equivalent constructed using a Brownian bridge (see, e.g., (Glasserman, 2003)) for comparison. The blue line is the observed (realized) closing price history of the S&P 500 between December 31, 1927 and August 14, 2020, a period over which the index returned 5.8% annually in nominal terms (i.e., ignoring dividends and without adjusting for inflation). The grey line is an artificial series sharing the same terminal values. Note that while both series have the same nominal realized returns, the artificial series accrues its returns in a much more predictable fashion.

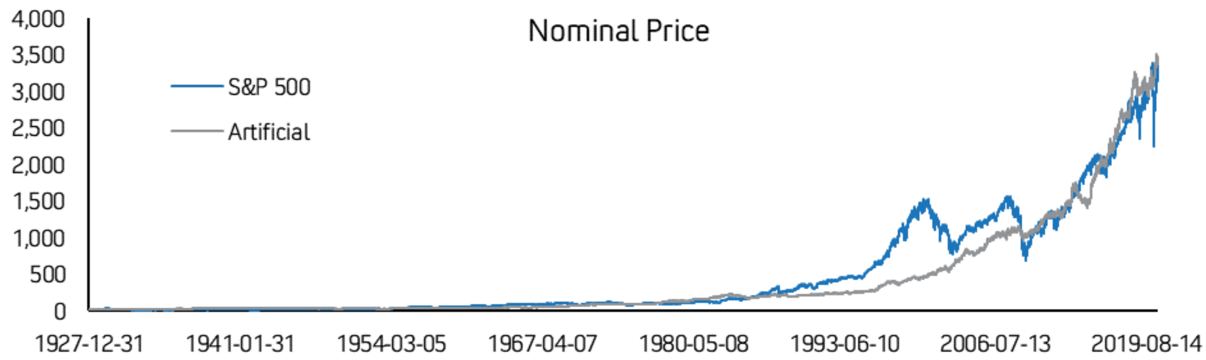

Figure 1: The S&P 500 returned 5.8% annually between December 31, 1927 and August 14, 2020, with a realized volatility of 19%. An artificial, low-volatility series, returns the same 5.8% annually but with a volatility of 10% is shown for comparison.

The respective realized (lognormal) volatilities over the observed period are 19% and 10%. Roughly speaking, the magnitude of daily actual returns was 40% larger than of returns associated with the artificial series. Furthermore, the realized (higher volatility) series exhibits far more extreme observations. Both of these features are illustrated in Figure 2.

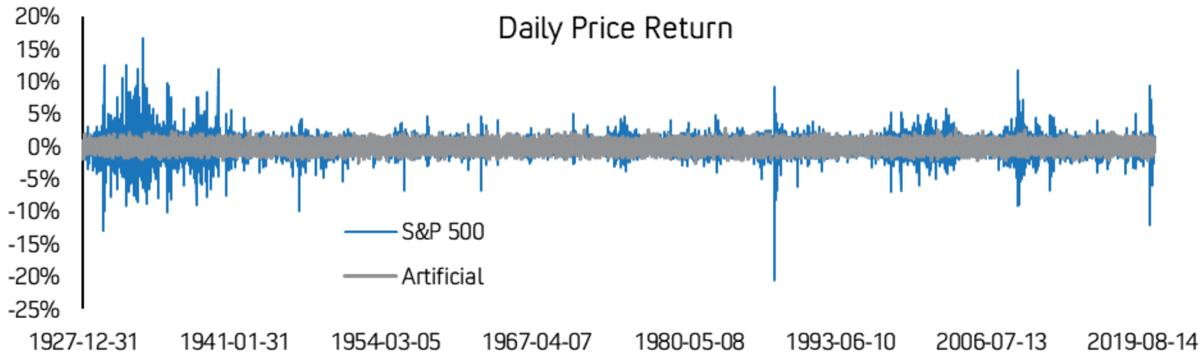

Figure 2: The daily price returns of the artificial low volatility investment are less extreme than those of the observed daily returns of the S&P 500.

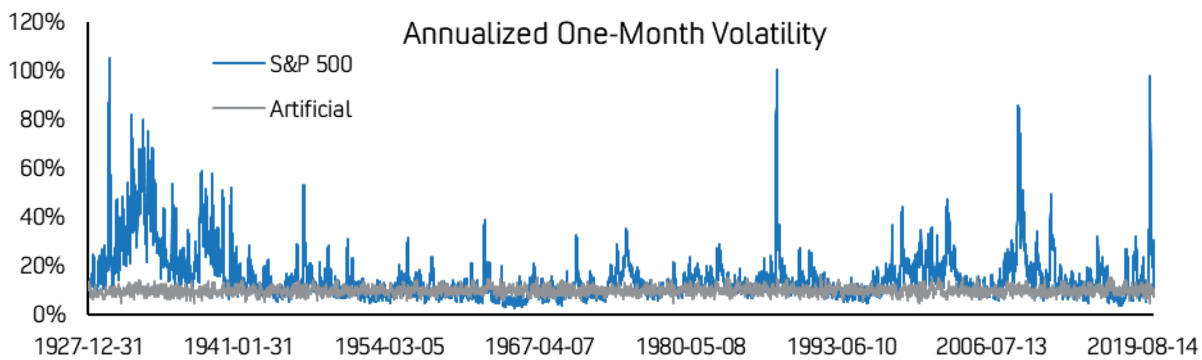

Figure 3: Realized volatility represents the observed dispersion of asset returns.

Figure 3 depicts one-month realized volatility of the S&P 500 calculated at the close of each trading day using the previous 21 trading days. Note that while returns can be positive or negative, volatility is strictly non-negative. Furthermore, large values of volatility coincide with periods of large absolute returns (Figure 2) and vice versa.

Implied Volatility

In options markets, volatility is essentially a price. The intuition behind this is that, other things being equal, the option to buy or sell an asset is increases in value with increasing dispersion of its price movements.

Assuming the Black-Scholes-Merton model (Black & Scholes, 1973; Merton, 1973) as a quoting convention, the price of an option is completely determined by its volatility. Therefore, option prices can be used to imply an expected volatility for the underlying asset. Prior to the stock market crash of 1987, assets' implied volatilities were approximately constant, consistent with this framework. That is, while many options may have traded on an individual equity, the underlying equity's volatility implied by each was roughly the same. Since then, however, volatility has exhibited a so-called smile, in which options with strikes far away from the current price of the asset exhibit higher implied volatilities than those near-the-money (Bates, 2008). Consequently, the asset volatility implied depends on the option(s) used.

CBOE VIX Index

The Chicago Board Options Exchange (CBOE) Volatility Index (VIX) (CBOE Volatility Index, 2019) is a broad measure of investors' (and speculators') expectation of annualized volatility of the S&P 500 and, by extension, U.S. equities over the next 30 days. It is a weighted function of implied volatilities of out-of-the-money[1] index options with maturities between 23 and 37 days and it is intended to capture the expected annualized standard deviation of log-returns over the next 30 calendar days (ibid). Although realized and implied volatility are seldom equal, they move coincidently because they respond to the same events (see Figure 4).

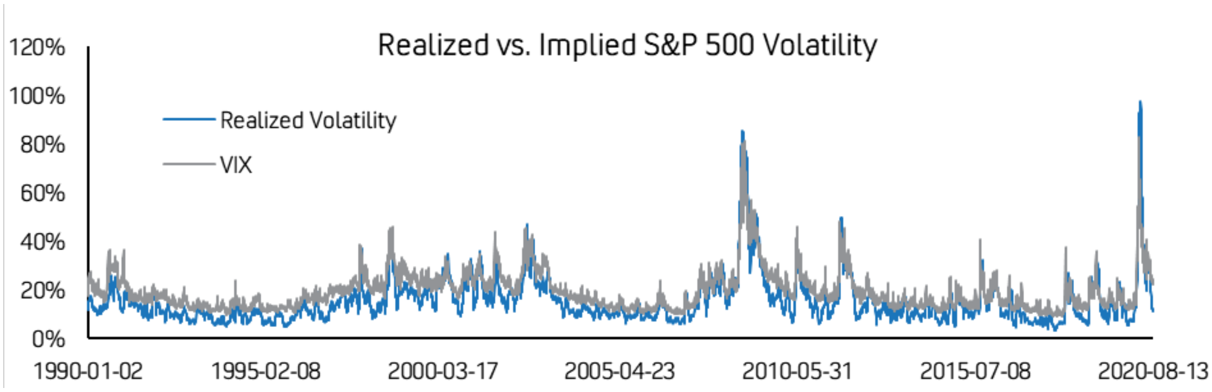

Figure 4: Realized and implied volatilities of the S&P 500 are similar but not equal.

There are two interesting conclusions that can be drawn from Figure 4. First, although realized and implied volatilities move in tandem, implied volatility (in the form of the VIX) is often larger than realized volatility. Indeed, since January 2, 1990, the VIX has closed higher than one-month realized volatility almost 90% of trading days. Second, although implied volatility is a forward-looking measure based on the prices at which market participants are willing to buy or sell options, it is influenced considerably by recent realized (historical) volatility.

Figure 5 plots the VIX against realized volatility; the dashed line is the 45-degree line. While there is a pronounced linear relationship, it is clear the relationship is not 1:1. Upon further reflection, this is not surprising, since even if realized volatility were identically zero, no market participant would sell an option for nothing. Therefore, even in the calmest economic conditions, implied volatility must be strictly positive to preclude zero-dollar option prices. Likewise, while the VIX often reacts strongly to sizeable

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[1] An option is out-of-the-money if the underlying price of the asset makes exercise unfavorable from the perspective of the option holder. Consequently, the only contribution to the option's value is the future volatility of the asset.

increases in realized volatility, there appears to be a level that sellers do not expect future volatility to exceed, suggesting that the slope of the line is less than one. A linear regression of VIX on realized volatility results in an intercept of approximately 8, a slope of 0.7 and a correlation of almost 90%.

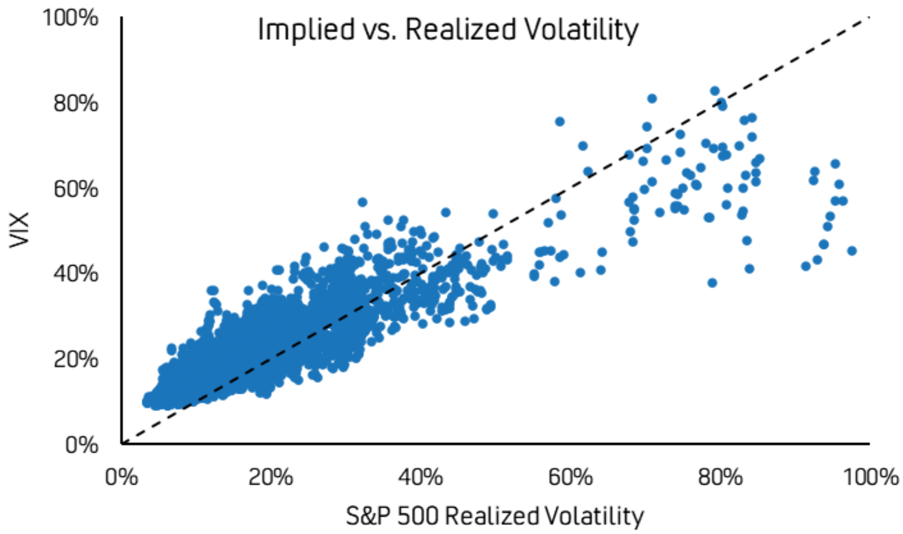


Figure 5: Realized and implied volatility exhibit a strong linear relationship.

A number of stylized behaviors was evident during the first half of 2020. Through January and most of February, as equity valuations increased steadily, realized and implied volatilities remained low and moved in concert. Although realized volatility was under 7% for almost two weeks, VIX was always above 12, reflecting the minimal level of risk option writers were willing to assume no matter how calm markets appeared. Conversely, while the VIX the VIX recorded its highest ever closing value (82.69) on March 16, realized volatility continued to rise for three weeks, peaking at 98% on April 6. Between March 17 and May 5, inclusive, realized exceeded implied volatility.

VIX Futures

The VIX itself cannot be bought or sold directly. This is perhaps not surprising considering Figure 4. Were it possible to trade VIX, speculators could buy at most levels, sell during the next spike, and realize a sizeable profit. Similarly, traders would be wary of selling at any but the highest levels, knowing with virtual certainty that an increase will come eventually. Clearly, the series in Figure 4 is not governed by the usual economics of supply and demand.

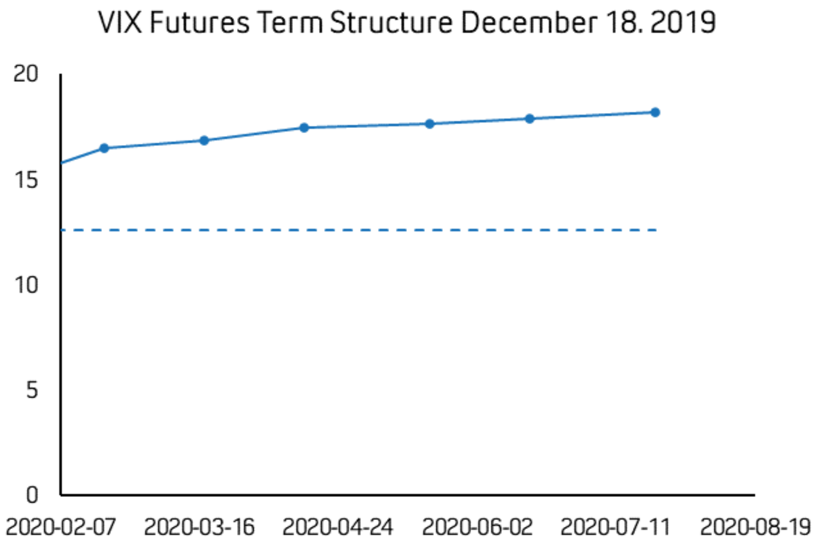
Nevertheless, it is possible for an investor to create synthetic exposure to the VIX by replicating its construction using S&P 500 options. The complexity of such an approach notwithstanding, this is an inefficient method of creating volatility exposure because an options portfolio has price sensitivity to other risk factors in addition to the underlying asset's volatility, requiring continual rebalancing—at considerable expense—to manage. Most significantly, sellers of out-of-the-money options bear price risk because movements in the underlying equity index will cause some of the sold options to increase in value, and the convexity of options prices makes it likely that these unfavorable price increases will not be offset fully by price declines of other short options moving further out-out-the-money. Buyers of options, on the other hand, suffer from time decay. Because the price uncertainty of the underlying asset decreases as the option nears maturity, the value of a long option position does as well. Thus, during relatively calm market conditions, the value of the buyer's options will decline steadily over time even if volatility remains unchanged.

Recognizing demand for volatility exposure, the CBOE introduced monthly-settled VIX futures contracts on March 26, 2004, listing contracts expiring in May and June of that year. As volume increased, the CBOE has listed an increased number of contracts, most recently weeklys beginning on October 8, 2015 (Chicago Board Options Exchange, 2017). Since 2008, futures contracts have been listed for at least the nearest seven serial months. These futures offer direct exposure to the VIX without the associated sensitivity to underlying price movements or time decay. (Although there is no element of time decay in the sense that there is with options, there is a time-dependency as futures maturing on different dates often trade at different, though related, prices. This is discussed further in the next subsection.) For the sake of completeness, we note that options on the VIX have traded since 2006 (VIX Options, 2019), although they, too, are inefficient as they are sensitive to the volatility-of-volatility and, like options on the S&P 500, suffer time decay. Options on the VIX are relevant to this discussion only inasmuch as they provide a means to infer a fair value for futures, a topic that will be discussed later.

Term Structure of Volatility Futures

Typically, the term structure of VIX futures is upward sloping (referred to as being in *contango*) as in Figure 6. Historical settlement prices (Chicago Board Options Exchange, 2020) show that weighted portfolios of the nearest three monthly contracts[2] exhibited this behavior 83% of the time between April 22, 2008 and August 14, 2020. Less often—and particularly in times of market stress—the futures curve exhibits a downward slope (referred to as *backwardation*) as illustrated in Figure 7.

[2] Specifically, a one-month constant maturity portfolio consisting of the first and second contracts and a two-month constant maturity portfolio consisting of the second and third contracts exhibited this relationship.



VIX	12.58
Month	**Settle**
Jan	14.825
Feb	16.425
Mar	16.825
Apr	17.425
May	17.575
Jun	17.875
Jul	18.125

Figure 6: The VIX Futures curve December 18, 2019 exhibited a typical upward sloping relationship between successive contracts. This so-called contango occurs approximately 85% of trading days. The dashed line represents the VIX.

While trading a futures contract can be viewed as taking a view on the *absolute* change in the level of volatility, the term structure (a representation of *relative* prices) has a significant influence on returns. For example, suppose an investor viewing the prices in Figure 6 felt that VIX (12.58) was likely to rise in the next month and therefore bought a January 2020 futures contract. On January 22, when that contract expired the VIX, indeed, had risen to 12.91 (+2.3%), however, the futures contract cost 14.85[3] to open. Despite having correctly predicted the direction of the VIX, in this example, the investor would have lost 13.1%!

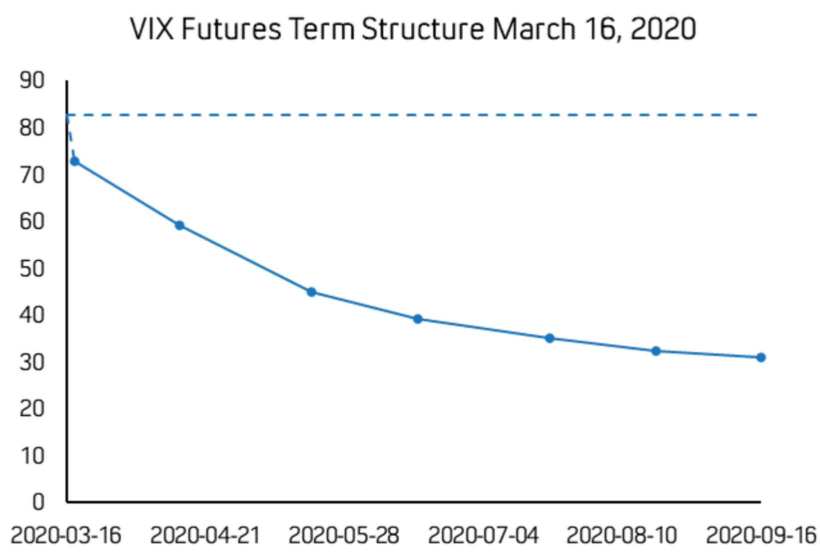
The impact of term structure is persistent and structural. To illustrate, consider another investor who felt that VIX would fall, and expressed that view by constructing a short portfolio consisting of only the first and second expiring contracts and with a weighted expiration of one month. On December 18, 2019, this short portfolio would have consisted solely of January contracts. Each day thereafter a fractional amount of the January contracts would be closed and replaced by an equal dollar amount of February contracts, such that by January 22, 2020 (the day the January contract expired), the portfolio would have consisted entirely of February contracts.

If the term structure did not change, a daily rebalanced short portfolio would have returned 10.4% if held for one month. This phenomenon is often incorrectly attributed to daily price changes in the contracts as they "roll down the curve." However, because the dollar amount is preserved at each rebalancing, this cannot be true. Although there are daily price fluctuations, the reason for the profitability of the short portfolio is that each daily rebalancing reduces the total number of short contracts over time. For example, on December 19, 2019 when one January contract is closed at 14.80, only 0.90 February

[3] VIX futures price increments are $0.05, and settlement is determined as the average of the bid and ask prices. Therefore, to open a long contract at the close a speculator would have needed to pay the ask price of $14.85.

contracts are required (1 × 14.80 = 0.90 × 16.45) to preserve the same notional amount invested. Therefore, after a month passes, the number of contracts held would be 10.4% less than when the portfolio was opened.

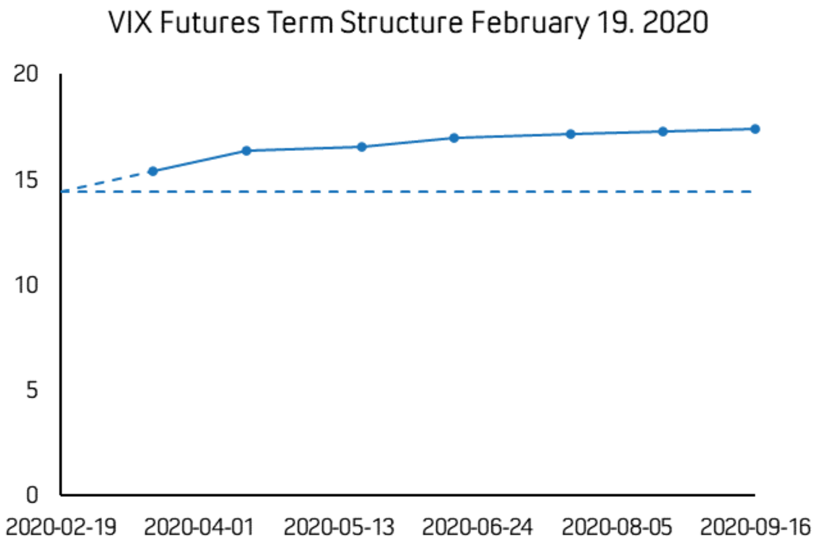
While the term structure of VIX futures is often upward sloping, it will, at times, become inverted as in Figure 7. Whereas an upward sloping term structure provided a tailwind for a short portfolio and a headwind for a long portfolio, a downward sloping term structure results in the opposite effect.



VIX	82.69
Month	**Settle**
Mar	72.625
Apr	59.150
May	44.875
Jun	38.950
Jul	34.975
Aug	32.175
Sep	30.875

Figure 7: The VIX Futures curve on March 16, 2020 exhibited an atypical downward sloping relationship between successive contracts. This so-called backwardation occurs approximately 15% of trading days. The dashed line represents the VIX.

Other things being equal, portfolios consisting of long futures contracts will benefit from backwardation, and portfolios consisting of short futures positions will be harmed by it. However, a majority of the variability in futures returns is explained by the change in the index itself and the (known) number of trading days to maturity, with futures prices following VIX more closely as they near expiration (Chicago Board Options Exchange, 2017). For instance, the term structure on February 19, 2020 (Figure 8) is qualitatively similar what it had been on December 18, 2019 (Figure 6). However, while a daily rebalanced one-month short position in VIX futures opened in December would have returned over 10%, a similar position opened in February would have lost 87%, as VIX rose from 14.38 to 76.45 between the expiration of the February and March futures.



VIX	14.38
Month	**Settle**
Mar	15.375
Apr	16.325
May	16.525
Jun	16.925
Jul	17.125
Aug	17.225
Sep	17.375

Figure 8: Although the term structure of VIX futures on February 19, 2020 favors a short position in futures, absent other information, such a position would have experienced a sizeable loss due to the increase in spot VIX over the following month.

The previous examples illustrate that there are two ways to profit from trading volatility futures. In calm or rising markets, short futures positions can return 10% monthly, essentially monetizing the term structure. Conversely in declining markets, the influence of rising VIX overwhelms the contribution of the term structure, and long positions can profit significantly. To wit, a daily rebalanced long position opened on February 19, 2020 would have gained 413%. Ideally, an investment strategy would hold a short position in VIX futures most of the time to exploit contango and take long positions when data suggest an upcoming increase in volatility. Because there are two sides to each derivatives transaction, there is, likewise, the potential for losses of the same magnitude.

Other Volatility Derivatives

Although the focus of this discussion is developing trading strategies using VIX futures, the market for volatility derivatives continues to expand. In the United States, the CBOE now lists volatility futures and options on equity indices, individual large-cap equities, commodities, and currencies. Likewise, in Europe, Eurex lists futures and options on the VSTOXX (the VIX analog corresponding to the STOXX 50 index). Vernon Capital Partners demonstrated that trading strategies described herein extend to European volatility derivatives (Annis, 2020).

Empirical Properties of VIX and VIX Futures

In this section we detail empirical properties of volatility, the VIX and VIX futures. The line plot of daily closing values of the VIX since its inception in 1990 (Figure 9) illustrates two well-known phenomena of implied volatility: mean reversion and jumps. Each is discussed in turn below, as are their implications.

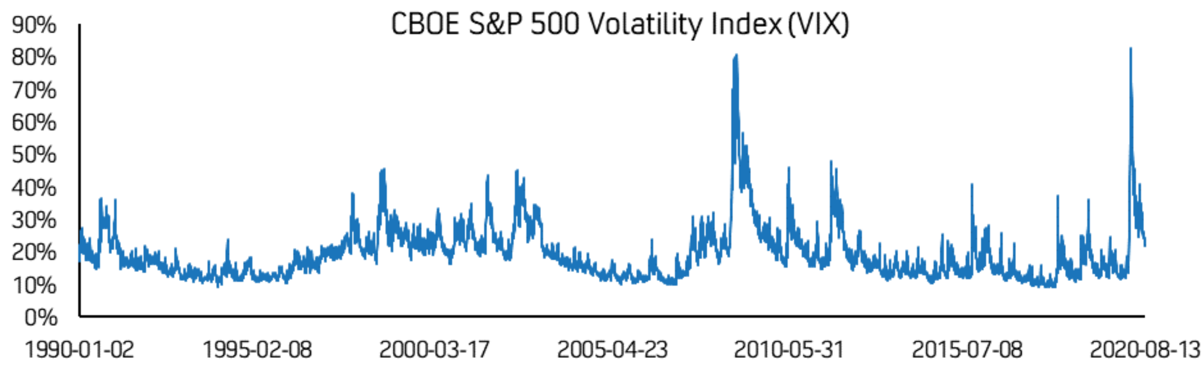


Figure 9: The VIX exhibits mean-reverting behavior over the medium- to long-term and can exhibit jumps in the short-term.

Mean Reversion

Mean reversion, or regression to the mean, is a property of data series that implies the further removed an observation is from its expectation, the greater the likelihood subsequent observations will return toward that expectation. It is often associated with the concept of bounded outcomes and was observed in the context of genetics as early as the late 19th century (Galton, 1886). In the context of finance, the key insight is that when a mean reverting series deviates markedly from its long-term mean, it is more likely to return than it is to become further removed.

Fitting a Vasicek (1977) model to the VIX time series suggests a mean value of 19 and a half-life of 30 trading days. Therefore, ignoring for the moment the influence of jumps (which will be discussed further in the following subsection), if the VIX closes at 29 today and assuming five trading days per week, it would be expected to be 24 by its close six weeks from now. Fernandes et al. (2014) confirm that the VIX and its predecessor, VXO, exhibit significant mean-reversion over time horizons as short as one week.

Jump Diffusion

In addition to its tendency to vary about a long-term level of roughly 20, Figure 9 shows that the VIX also exhibits large jumps. Todorov and Tauchen (2011) confirm the observation that jumps are non-negligible (i.e., that a pure diffusion process is inadequate to describe the stochastic behavior of the VIX completely) and occur synchronously with large equity moves. This behavior is more noticeable in the time series of daily VIX returns (Figure 10) instead of daily values.

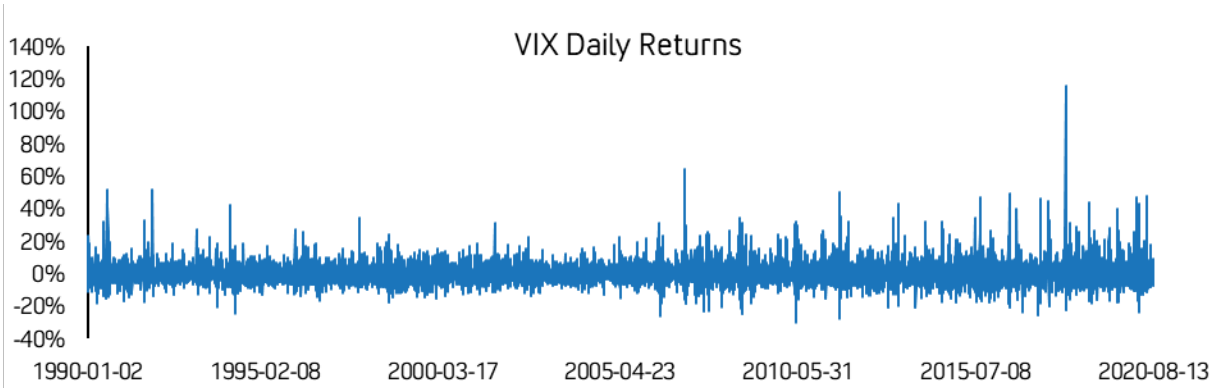


Figure 10: The VIX exhibits experiences periodic jumps, with upward jumps more common than downward ones.

Asymmetric Response to Realized Volatility

It is well-known empirically that the VIX is prone to sharp increases when equity markets decline, although its response to significant equity gains is more muted. For this reason, it is sometimes referred to as a "fear gauge." This is at first counterintuitive because volatility is directionless, and therefore it might be expected to react to the magnitude of equity movements irrespective of direction. This would be true if the options market consisted solely of individual speculators equally likely to take either side of a trade. This is not the case, though, as large institutions and asset managers with long exposure to equity markets use options on the S&P 500 (and other indices) to hedge, increasing demand for puts asymmetrically (Rhoads, 2012).

The VIX, therefore, is often biased toward downward movements because the options contributing to its calculation are disproportionately puts. To illustrate, consider the calculation of the index intraday on August 17, 2020. There were 925 distinct out-of-the-money S&P 500 options maturing on September 11 or September 18 relevant to the calculation of the VIX. Of those, 77% (710) were puts; put volume (39,116) was 87% of total volume (44,924); and put open interest (2,080,431) was 90% of total open interest (2,320,279) (Chicago Board Options Exchange, 2020).

This overweighting of puts relative to calls is typical. In fact, between July 6, 2010 and October 4, 2019, more puts were traded than calls on 99% (2,306) of trading days (2,330) (S&P 500 Index Volume and Put/Call Ratio Archive, 2020); the median ratio of put-to-call volume was 1.71 over the same period (ibid), meaning 71% more puts were traded than calls (see Figure 11).

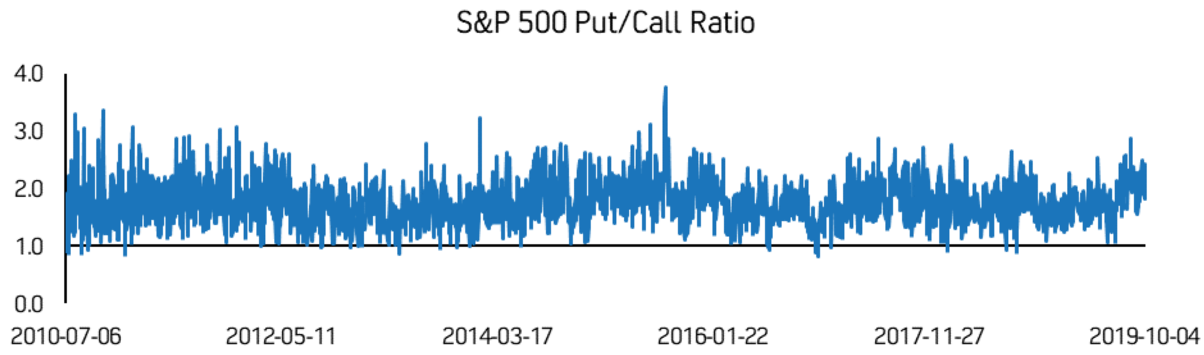


Figure 11: The ratio of puts to calls traded on the S&P 500 exceeds parity 99% of trading days.

A sharp decline in equity prices would inflate the prices and, consequently, the volatility of the larger number of put options, thereby producing a commensurately sharp rise in the VIX. Conversely, a sharp move upward by equity prices would decrease the price of those puts resulting in a lower value of the index despite a likely rise in price of the call options.

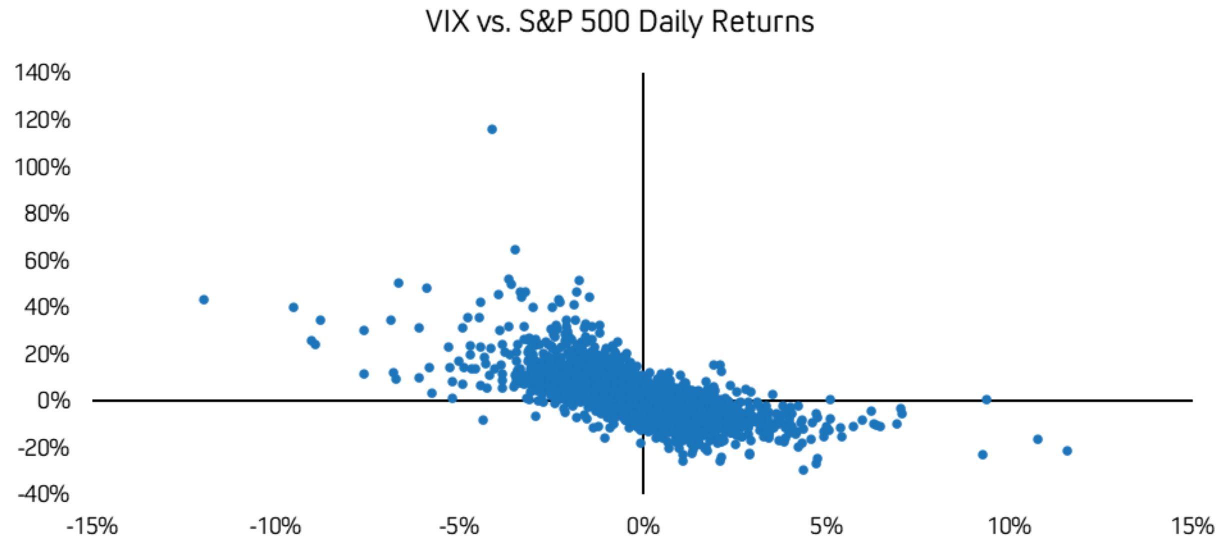


Figure 12: Daily returns for the VIX are inversely related to the S&P 500.

The impact of this asymmetry is illustrated in Figure 12 which shows the negative correlation between daily returns in the S&P 500 and the VIX. The sign of the returns differs for 79% (6,064) of trading days (7,715) between January 2, 1990 and August 14, 2020, but over the same period on days in which the S&P 500 moved by more than 1% in either direction, VIX moved opposite in 93% (1,537) of days (1,651). The scales of the respective axes in Figure 12 are worth noting, as VIX returns are much larger in magnitude than equity returns. Over the period tested, the VIX returned -4.2 times the amount of the S&P 500 per trading day.

VOLATILITY AS AN ASSET CLASS

While the view of volatility derivatives as an equity hedge is specious, long-term returns tend to be uncorrelated to equities, commodities, foreign currencies, and interest rates. In that regard, volatility derivatives can provide a degree of portfolio diversification (Szado, 2009). In fact, the notional value of open interest in volatility derivatives has increased steadily over the last two decades (Figure 13). This section examines factors influencing the value, and consequently estimate the price, of volatility futures.

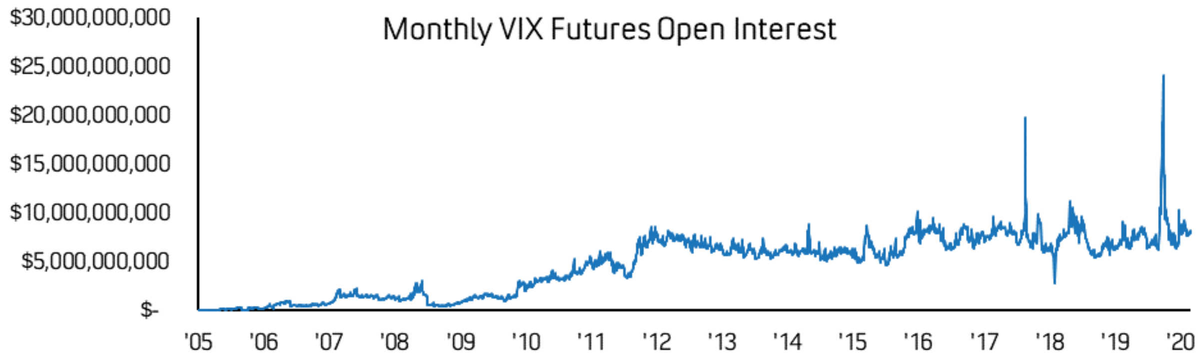


Figure 13: Open interest in VIX futures has increased steadily since 2005.

TERM STRUCTURE

The strong negative correlation between *daily* returns of S&P 500 and VIX (see Figure 12) leads some retail investors to "hedge" their equity exposure with long positions in the VIX, however the utility of doing so is suspect. While it would have been possible, with the benefit of extraordinarily timing, to profit from this inverse relationship over short periods, a constant maturity portfolio of long VIX futures held between April 22, 2008 and August 14, 2020 would have lost 99.9% of its value. In large part, this is due to contango in the term structure, which can steadily erode up to 10% of the value of the portfolio per month. Figure 14 compares monthly returns of a long portfolio of VIX futures to the VIX itself. Note that by design, a one-month maturity futures portfolio is intended to mimic the VIX (which is a 30-day measure of implied volatility). The futures portfolio returns roughly 50% of what the index does minus 3% per month. Since the VIX is mean reverting, an intuitive, albeit mathematically imprecise, way to interpret this relationship is that when VIX is at a typical level, over a long horizon, the return will be -3% per month (-30% annually).

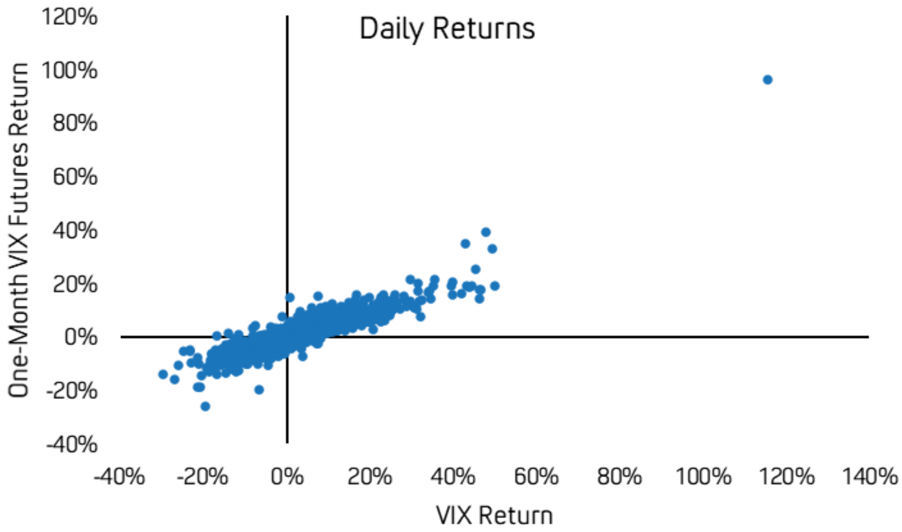


Figure 14: Monthly returns of the VIX and a daily rebalanced long one-month maturity portfolio of VIX futures illustrates futures movements are less pronounced than index movements and, on average, lose 5% per month when the index is unchanged.

Research published by the CBOE (2017) confirms the responsiveness of VIX futures to changes in the index itself decreases with increasing time to expiration. These results are expected considering the demonstrated mean reversion of the index, which can be viewed as a futures contract on itself with no time remaining until expiration. Because it is mean reverting, large deviations from typical levels will dissipate over time, with longer time periods allowing for more mean reversion to occur.

While the term structure of volatility does not provide a mechanism to value contracts *per se*, it does allow estimation of a constant maturity portfolio's returns assuming no change in the value of the index itself. This estimate can be viewed as a gauge of favorability of long or short futures portfolios.

Fair Value of VIX Futures

Much of financial mathematics is built on the idea of no-arbitrage pricing. Put simply, it is often possible to re-create an asset synthetically with an appropriately constructed portfolio of related securities. For example, a portfolio of regularly maturing zero-coupon bonds provides identical cash flows to a single coupon-paying bond. Likewise, a European call option can be created using its corresponding put option, the underlying asset, and a government bond. See, e.g., Hull (2016) for a complete discussion of arbitrage and its implications for pricing and hedging financial instruments.

When this sort of replication is possible, the prices of the asset and its replicating portfolio should be equal (or at least within the transaction costs incurred to buy one and sell the other). Traded options on the VIX and a statistical estimate of the volatility of volatility, following Dupire (2006) can be used to infer fair values of listed futures contracts. Often, futures trade near their derived fair values, however sizeable deviations offer opportunities to buy or sell contracts to exploit temporary mispricing. While this is likely true in the long term, it is worth remembering Keynes's warning that "markets can stay irrational longer than you can stay solvent."

VIX as a Forecast of Future Volatility

Recall the VIX is designed to measure 30-day expected volatility by inferring market participants' consensus from options prices. It is, therefore, intended as a forward-looking estimate of volatility. However, for it to be truly predictive, market participants must be able to anticipate events contributing to large equity price changes. This is a very strong assumption and one unlikely to be true. More likely, VIX reflects current market uncertainty and is influenced by recent realized volatility.

Recent research (Adhikari & Hilliard, 2014; Asness, 2017) supports this interpretation. The implication of VIX as a retrospective, rather than prospective, estimate of volatility is that realized volatility provides another method to value VIX and VIX futures.

Volatility Trading

Motivation

Two appeals of volatility as a distinct asset class are that it is mean-reverting and that it is dimensionless. By contrast, equities seldom demonstrate mean reversion are denominated in dollars (or other fiat currencies). This makes forecasting equity prices, for example, more difficult in two ways. First, not only does an accurate forecast require a reliable estimate of real economic value in the future, it must also account for the variable purchasing power of the reference currency. Shiller (2012) makes this argument in concluding that most—if not all—gains seen in (nominal) U.S. house prices over the long term are the result of dollar inflation rather than real price appreciation.

Conversely, it is much easier to forecast the level of the VIX over long horizons. The tendency for the index to exhibit mean reversion means that we can be 65% confident that twenty years from now the VIX will be between 15 and 25, for example, whereas similar statements about the S&P 500 or of commodities such as gold or crude oil are much more difficult to make. Furthermore, because it is dimensionless, no adjustment is required to normalize units over time. It is worth noting that even if one can feel confident about a long-term forecast about the VIX index, the index itself cannot be traded directly, and futures strategies carry significant risk of loss.

Intuition

A strategy combining futures term-structure, option-implied fair-values, mean-reversion, and trailing realized volatility may be used to estimate the following day's return of a portfolio of VIX futures. Each element provides some insight into likely near-term price movements. As illustrated in Figure 6 and Figure 7, the term-structure of futures influences returns significantly, and though movements in the underlying VIX explain more of the daily changes in futures, the impact of the term-structure is more predictable over longer time horizons. A futures contract's fair value above (respectively, below) its price suggests an impending rise (respectively, fall) to achieve an equilibrium. Similarly, when the VIX is trading above (respectively, below) its long-run average, the index is likely to revert toward its mean over time. Furthermore, although realized and implied volatilities can and do differ (recall Figure 4 and Figure 5), they are highly correlated, suggesting any significant dislocation between realized volatility and the VIX will dissipate over time.

Two critical elements of any strategy are position sizing, reflecting the relative attractiveness of the opportunity for that day, and appropriately chosen exit criteria intended to minimize large drawdowns. This is not to suggest that significant drawdowns can be avoided completely; rather their size and frequency can be limited to levels that make the overall strategy attractive from a risk-return standpoint. Note that for large notional values of invested capital liquidity constraints may limit the utility of stop-loss orders.

Pro Forma Performance

Beginning on May 1, 2019, Vernon Capital Partners has traded this strategy in a proprietary account. Figure 15 shows the pro forma account balance (net of 1.5% management fees, paid as 0.125% of monthly average assets under management and 17.5% incentive fee on new profits in excess of previous high-water marks paid quarterly). The pro forma total return since inception is 60%, which annualizes to 40%. Monthly performance is given in Table 1.

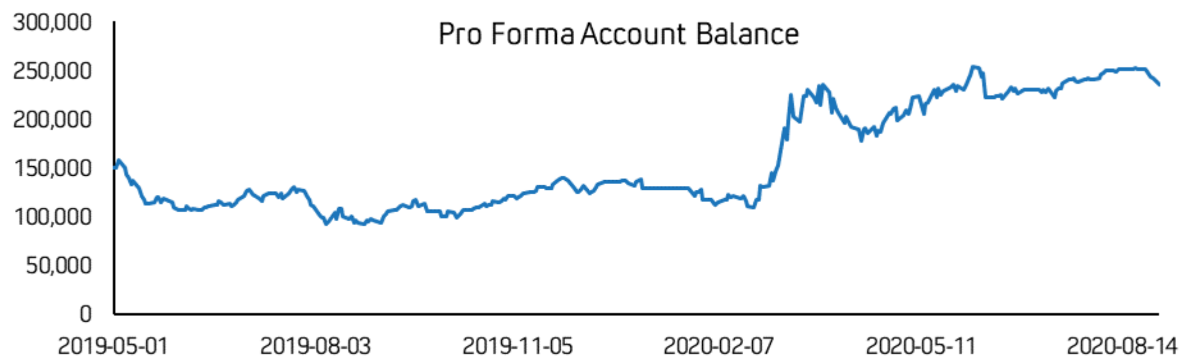


Figure 15: The proprietary trading account has returned over 40% annualized since inception net of hypothetical fees.

	2019	2020
Jan		-13.5%
Feb		+11.3%
Mar		+59.2%
Apr		+2.4%
May	-28.9%	+11.5%
Jun	+10.4%	-2.0%
Jul	-0.4%	+5.2%
Aug	-17.5%	-2.4%
Sep	+8.3%	
Oct	+12.0%	
Nov	+17.3%	
Dec	-0.9%	
Total	-9.2%	+75.9%

Table 1: The proprietary account gained 60% net of hypothetical fees since inception.

PAST PERFORMANCE IS NOT NECESSARILY INDICATIVE OF FUTURE RESULTS.

While trading results have been encouraging, the program may experience sizeable drawdowns. This is evident in the performance of the proprietary account in its first few months. Figure 16 illustrates a month-end peak-to-trough decline of 35% between May and August 2019.

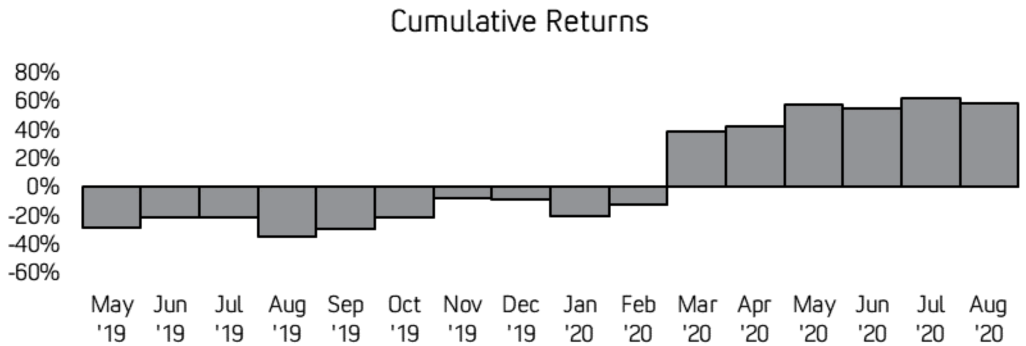


Figure 16: Cumulative month-end returns illustrate the non-linearity of returns.

Other Considerations

Potential Tax Advantages of Futures for U.S. Investors

Under Section 1256 of the Internal Revenue Code, investment gains from futures contracts are taxed at a blended rate equal to 60% of the long-term and 40% of the short-term capital gains rates (United States

Department of the Treasury, 2018). For investors in the 37% marginal tax bracket, this means that capital gains from futures trading are taxed at 28.6%, irrespective of how long the positions were open.

Volatility of Volatility

Coinciding with the introduction of options on the VIX, the CBOE has computed a VIX of VIX (VVIX) index (Chicago Board Options Exchange, 2012). Figure 17 provides coincident time series of the VIX and the VVIX. While jumps tend to be synchronous, the absolute level of VVIX is markedly higher than the level of the VIX. This is expected because the daily returns of the VIX are, in general, over four times those of the S&P 500. In fact, while the typical level of the VIX is below 20, the typical level for VVIX is closer to 90.

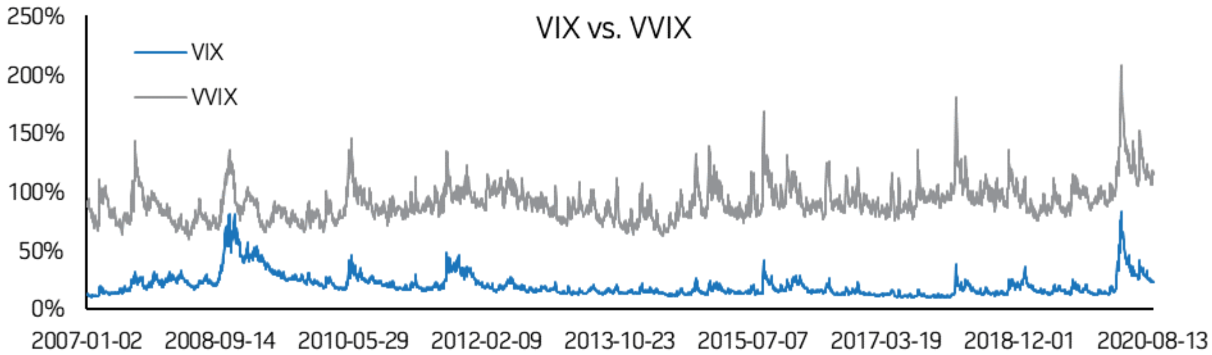


Figure 17: The volatility of the VIX (VVIX) is significantly higher than the level of VIX itself.

This increased volatility of volatility (when compared to volatility of broader equity markets) underscores the frequency with which large moves occur. It is imperative to understand that any strategy designed to profit from trading volatility derivatives will, at times, suffer from downside volatility. In one respect, this is an advantage for systematically managed futures portfolios because undisciplined and emotional trading degrades performance (Barber & Odean, 2000; Meyer, Schroff, & Weinhardt, 2014) of the individual speculators likely to be on the other side of our trades.

About Vernon Capital Partners

Vernon Capital Partners is a registered Commodity Trading Advisor specializing in volatility derivatives and related instruments. Our investment philosophy has evolved over two decades spent leading capital markets analytics and risk management teams. Our proprietary, inferential models are designed to identify profit opportunities by quantifying market inefficiencies in derivatives. We aim to be uncorrelated with equities, bonds, commodities and volatility itself over intermediate- to long-term time horizons.

David H. Annis, Ph.D.

David H. Annis, Ph.D. is Chief Investment Strategist and a Founding Principal of Vernon Capital Partners. He most recently led Wells Fargo's Comprehensive Capital Analysis and Review (CCAR) and Central Counterparty (CCP) analytics team and was previously Head of Trading and Risk Model Validation, where his team was responsible for validation and oversight of over 100 models used for evaluating and managing market and counterparty risk arising from derivative transactions.

Before transitioning to finance, Dr. Annis taught at the Naval Postgraduate School where, as a member of the Operations Research faculty, he held a Level 3 (Top Secret) Department of Defense security clearance and published over a dozen peer-reviewed research papers. He devised a long-range surveillance technique using signal processing and pattern recognition at Los Alamos National Laboratory and began his career at Pratt & Whitney designing compressor hardware for military jet engines.

Dr. Annis earned his Ph.D. and M.S. in Mathematical Statistics from Purdue University, holds engineering degrees from Purdue and the University of Florida and earned his MBA with Distinction from Warwick Business School (UK), researching investment strategies based on European volatility derivatives.

Mark P. Kust, CFA

Mark P. Kust, CFA is Chief Operating Officer and a Founding Principal of Vernon Capital Partners. He is a CFA Charterholder and PRMIA Professional Risk Manager (PRM) and most recently led the CRO Model Risk and Governance function at the World Bank Group. Mark was previously Head of Model, Market and Strategic Risk Management at Brighthouse Financial where he designed, implemented and led that firm's Model Risk Management function. He co-led Model Validation for Ally Financial after leading Capital Markets Model Validation Analysis and Support for Wells Fargo. While at Wells, Mark held a variety of roles on the Commodity Derivatives Trading Desk, spanning Business Management, Technology, Structured Pricing, Quantitative Analysis and Risk Management.

Prior to Mark's tenure in financial services, he was a technology executive. He led Dell's Intranet Standards and Applications groups in the late 1990s, and was co-founder and Chief Information Officer for ChemCodes Inc., an entrepreneurial life sciences firm. At ChemCodes Mark designed novel pattern recognition algorithms for processing high throughput mass spectroscopy signals and designed and implemented a full lifecycle Laboratory Information Management System (LIMS).

Mark began his career at Los Alamos National Laboratory and has graduate degrees in engineering and mathematics from Stanford University, and an MBA from Duke University.

REFERENCES

Adhikari, B. K., & Hilliard, J. E. (2014). The VIX, VXO and Realised Volatility: A Test of Lagged and Contemporaneous Relationships. *International Journal of Financial Markets and Derivatives*, 222-240.

Annis, D. H. (2020). The Volatility Risk Premium in European Equities: Evaluating VSTOXX Futures Strategies in Retail Investment Portfolios. *Warwick Business School Dissertation*. Coventry, United Kingdom: University of Warwick.

Asness, C. (2017). *Please Stop Talking About the VIX So Much*. Retrieved from https://www.aqr.com/Insights/Perspectives/Please-Stop-Talking-About-the-VIX-So-Much

Barber, B. M., & Odean, T. (2000). Trading Is Hazardous to Your Wealth: The Common Stock Investment Performance of Individual Investors. *Journal of Finance*, 773-806.

Bates, D. S. (2008). The Market for Crash Risk. *Journal of Economic Dynamics and Control*, 2291-2321.

Bekaert, G., & Hoerova, M. (2014). The VIX, the Variance Premium and Stock Market Volatility. *Analysis of Financial Data, Journal of Econometrics*, 181-192.

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 637-654.

Chicago Board Options Exchange. (2012). *VVIX - Term Structure.* Retrieved from http://www.cboe.com/framed/pdfframed?content=/micro/vvix/documents/vvix-termstructure.pdf&section=SEC_OPTIONS_PRODUCTS&title=VVIX+-+Term+Structure

Chicago Board Options Exchange. (2017). *VIX Weeklys Options and Futures.* Retrieved from https://www.cboe.com/micro/vix-weeklys/pdf/vix-weeklys-options-and-futures-fact-sheet.pdf

Chicago Board Options Exchange. (2019). *CBOE Volatility Index.* Retrieved from https://www.cboe.com/micro/vix/vixwhite.pdf

Chicago Board Options Exchange. (2019). *VIX Options*. Retrieved from http://www.cboe.com/products/vix-index-volatility/vix-options-and-futures/vix-options

Chicago Board Options Exchange. (2020, August 17). *Quote Table Download*. Retrieved from http://www.cboe.com/delayedquote/quote-table-download

Chicago Board Options Exchange. (2020). *S&P 500 Index Volume and Put/Call Ratio Archive*. Retrieved from CBOE Volume & Put/Call Ratios: http://www.cboe.com/data/historical-options-data/volume-put-call-ratios

Chicago Board Options Exchange. (2020). *VIX Futures Historical Data*. Retrieved from https://markets.cboe.com/us/futures/market_statistics/historical_data/

Dupire, B. (2006). Model Free Results on Volatility Derivatives. *Unpublished Manuscript, Bloomberg LP*.

Fernandes, M., Medeiros, M. C., & Scharth, M. (2014). Modeling and Predicting the CBOE Market Volatility Index. *Journal of Banking and Finance*, 1-10.

Galton, F. (1886). Regression Towards Mediocrity in Hereditary Stature. *Journal of the Anthropological Institute of Great Britain and Ireland*, 246-263.

Glasserman, P. (2003). *Monte Carlo Methods in Financial Engineering (Stochastic Modelling and Applied Probability (53)).* New York: Springer.

Hull, J. C. (2016). *Fundamentals of Futures and Options Markets* (9th ed.). Boston: Pearson.

Merton, R. C. (1973). Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science*, 141-183.

Meyer, S., Schroff, S., & Weinhardt, C. (2014). (Un)skilled Leveraged Trading of Retail Investors. *Financial Markets & Portfolio Management*, 111-138.

Rhoads, R. (2012). *Trading with the VIX Put/Call Ratio.* Retrieved from Chicago Board Options Exchange: http://www.cboe.com/publish/oicommentary/rr_11_24_11.pdf

Shiller, R. J. (2012, April 14). Why Home Prices Change (or Don't). *New York Times*, p. 6.

Szado, E. (2009, August). *VIX Futures and Options – A Case Study of Portfolio Diversification During the 2008 Financial Crisis.* Retrieved from https://www.cboe.com/micro/vix/VIXFuturesOptionsUMassFull.pdf

Todorov, V., & Tauchen, G. (2011). Volatility Jumps. *Journal of Business and Economic Statistics*, 356-371.

United States Department of the Treasury. (2018). *Internal Revenue Service Form 6781: Gains and Losses From Section 1256 Contracts and Straddles.* Retrieved from https://www.irs.gov/forms-pubs/about-form-6781

Vasicek, O. (1977). An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics*, 177-188.